Bound States in the Vortex Core

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Abstract

The quasiparticle excitation spectrum of isolated vortices in clean layered d-wave superconductors is calculated. A large peak in the density of states in the "pancake" vortex core is found, in an agreement with the recent experimental data for high-temperature superconductors.

1 Introduction

The vortex core in classical type-II superconductors can be treated as the normal metal area with radius of the order of the coherence length \( \xi(0) \sim 10 \text{nm} \). The spectrum of the bound states near the Fermi surface, formed by the constructive interference between the incident and the Andreev reflected quasiparticles, is quasicontinuous (gapless). High-temperature superconductors (HTS) are layered, having a cylindrical Fermi surface, the superconductivity is of the strong and d-wave coupling type, and the vortex core radius is much smaller, \( \xi(0) \sim 1 \text{nm} \).

For a two-dimensional (2D) vortex and s-pairing, Rainer et al. have shown, using the Andreev quasiclassical theory in the analytical, and Eilenberger’s in the numerical part of their study, that bound states exist in the vortex core. Similar conclusions have also been obtained by Maki and coworkers for d-pairing, within the Bogoliubov-de Gennes approach.

In this paper, the Eilenberger quasiclassical equations, in the case of both s and d-pairing, are solved analytically, within the model considering the spatial variation of the order parameter in the vortex core as for a normal metal cylinder of radius \( r_c \sim \xi \). A crucial difference is found in the quasiparticle spectra below the bulk energy gap, between the classical superconductors with spherical Fermi surface, s-pairing, large \( \xi \), and HTS with cylindrical Fermi surface, d-pairing, small \( \xi \). Our results confirm that the quasiparticle density of states (DOS) has one large maximum, recently observed in YBCO by scanning tunneling microscopy.

2 Model of the Vortex Core

An efficient method for calculating local spectral properties is the quasiclassical theory of superconductivity, which gives the Eilenberger equations

\[
\begin{align*}
2\hbar \omega_n + \hbar \nu \cdot \left( \nabla - \frac{i e}{\hbar c} A \right) f &= 2\Delta g, \\
2\hbar \omega_n - \hbar \nu \cdot \left( \nabla + i \frac{2e}{\hbar c} A \right) f^\dagger &= 2\Delta^* g,
\end{align*}
\]

\[
\hbar \nu \cdot \nabla g = \Delta^* f - f^\dagger \Delta.
\]
Here \( g = g_{11}(r, v, \omega_n) \) and \( f = f_{11}(r, v, \omega_n) \) represent the normal and the anomalous Green function respectively, \( \Delta = \Delta(r, v) \) is the gap function, \( \omega_n = \pi k_B T (2n + 1), \) \( n = 0, \pm 1, \pm 2, \ldots \), are the Matsubara’s frequencies, and \( v \) is the Fermi velocity vector. The function \( f^\dagger \) is defined by \( f^\dagger (r, v, \omega_n) = f_{11}^*(r, -v, \omega_n) \). \( \Delta \) and \( f \) are connected by the self-consistency equation.

For a homogeneous and isotropic superconductor, solutions of the Eilenberger equations are

\[
\langle f \rangle = \frac{\Delta}{\varepsilon_n}, \quad \langle f^\dagger \rangle = \frac{\Delta^*}{\varepsilon_n}, \quad \langle g \rangle = \frac{\hbar \omega_n}{\varepsilon_n} \quad \left( \varepsilon_n^2 = |\Delta|^2 + (\hbar \omega_n)^2 \right).
\]

For a pancake vortex in \((r, \varphi)\) plane, denoting the coordinate along \( v \) by \( s \), and along \( h \times v \) by \( p \) (Fig.1.), in the gauge with real gap, Eqs. (1) and (2) can be rewritten in the form

\[
\begin{align*}
2\hbar \omega_n + hv \left( \frac{\partial}{\partial s} + \nu \frac{p}{l_H^2} + \nu \frac{p}{r^2} \right) f &= 2\Delta(r, \theta)g, \\
2\hbar \omega_n - hv \left( \frac{\partial}{\partial s} - \nu \frac{p}{l_H^2} - \nu \frac{p}{r^2} \right) f^\dagger &= 2\Delta(r, \theta)g,
\end{align*}
\]

\[
hv \frac{\partial}{\partial s} g = \Delta(r, \theta) (f - f^\dagger),
\]

where \( r^2 = p^2 + s^2 \) and \( l_H^2 = \hbar c/eh \).

For a normal metal cylinder and zero magnetic field, Eqs. (8)-(11) with \( p = 0 \), the solution is of the form

\[
f = \sum_i f_i(p)e^{\kappa_is}, \quad g = \sum_i g_i(p)e^{\kappa_is}.
\]

For \( r \leq r_c \), with \( \kappa_0 = 2\omega_n/\nu \),

\[
f = Fe^{-\kappa_0 s}, \quad g = G.
\]

For \( r > r_c \) and \( \kappa = 2\varepsilon_n/hv \)

\[
f = \langle f \rangle + \Phi_1 e^{-\kappa_0 s}, \quad g = \langle g \rangle + \Gamma_1 e^{-\kappa_0 s}, \quad \text{for} \ s > 0,
\]

\[
f = \langle f \rangle + \Phi_2 e^{\kappa_0 s}, \quad g = \langle g \rangle + \Gamma_2 e^{\kappa_0 s}, \quad \text{for} \ s \leq 0.
\]

Eqs. (8)-(11) imply

\[
\frac{\Phi_1}{\Gamma_1} = \frac{\Delta(\theta)}{\hbar \omega_n - \varepsilon_n}, \quad \frac{\Phi_2}{\Gamma_2} = \frac{\Delta(\theta)}{\hbar \omega_n + \varepsilon_n}.\]

Using the continuity condition for \( f \) and \( g \) at \( \pm s_0 = \pm \sqrt{r_c^2 - p^2} \), for \( r < r_c \) the normal Green function is

\[
G = \frac{\hbar \omega_n \cosh(\kappa_0 s_0) + \varepsilon_n \sinh(\kappa_0 s_0)}{\hbar \omega_n \sinh(\kappa_0 s_0) + \varepsilon_n \cosh(\kappa_0 s_0)}.
\]
For a vortex, approximating \( p/r^2 \) by \( p/r_c^2 \), the solution of Eq. (3)-(6) can be obtained from Eq. (13), by changing \( \omega_n \rightarrow \omega_n' \),

\[
G \approx \frac{\hbar \omega'_n \cosh(\kappa'_n s_0) + \varepsilon'_n \sinh(\kappa'_n s_0)}{\hbar \omega'_n \sinh(\kappa'_n s_0) + \varepsilon'_n \cosh(\kappa'_n s_0)},
\]

where

\[
\omega'_n = \omega_n + \frac{p v}{2} \left( \frac{1}{r_c^2} + \frac{1}{l_H^2} \right),
\]

and \( \kappa'_n = 2\omega'_n/v \). In this case, the magnetic flux quantization leads to \( \mathbf{1}, \mathbf{3} \)

\[
p_i = \left( i + \frac{1}{2} \right) \frac{\hbar}{m v}, \quad i = 0, \pm 1, \pm 2, \ldots
\]

Since for an isolated vortex \( l_H \gg r_c \), the direct influence of the field can be neglected, and the only relevant contribution is due to the screening supercurrent flow, \( \nu p v/2r_c^2 \) term in Eq. (15).

### 3 Bound States

Performing an analytical continuation of \( G \) by \( \hbar \omega_n \rightarrow -i E + \eta \), \( E \) being the quasiparticle energy with respect to the Fermi level, the retarded propagator \( g^R(E, p, \theta) \) is obtained. In terms of reduced variables \( E/\Delta_0 \rightarrow E \), \( \sqrt{\Delta^2(\theta) - E^2}/\Delta_0 \rightarrow \varepsilon \), \( \sqrt{E^2 - \Delta^2(\theta)}/\Delta_0 \rightarrow e \), \( p/r_c \rightarrow p \), \( 2s_0/\pi \xi_0 \rightarrow s_0 \), \( \xi_0 = h v/\pi \Delta_0 \) being the BCS coherence length, angle resolved partial DOS (PDOS) is obtained from \( N(E, p, \theta) = \text{Reg}^R(E, p, \theta) \).

For the normal metal cylinder

\[
N(E, p, \theta)/N(0) = \Theta\left(e^2\right) \frac{|E|e}{e^2 \cos^2(Es_0) + E^2 \sin^2(Es_0)} + \Theta\left(\varepsilon^2\right) \frac{\pi |\Delta(\theta)|}{\Delta_0} \delta(E \sin(Es_0) - \varepsilon \cos(Es_0)),
\]

where \( \delta \) is the Dirac function, \( \Theta \) is the step-function, and \( N(0) = m/2\pi \hbar^2 \) is the normal metal density of states at the Fermi surface for one spin orientation. For \( s \)-wave pairing, PDOS does not depend on \( \theta \), while for \( d \)-wave pairing, averaging over the cylindrical Fermi surface leads to

\[
N(E, p)/N(0) = \frac{1}{2\pi} \int_0^{2\pi} N(E, p, \theta)/N(0) d\theta = \\
= \frac{2}{\pi} \int_{\max(0, E^2 - 1)}^{E} \frac{|E|e^2 de}{\sqrt{E^2 - e^2} \sqrt{1 - E^2} - E^2} + \frac{(E \tan(Es_0) + |E \tan(Es_0)|)}{\sqrt{\cos^2(Es_0) - E^2}} \Theta(\cos^2(Es_0) - E^2).
\]

Finally, after spatial averaging over the cylinder area \( \pi r_c^2 \), DOS is

\[
N(E) = \frac{4}{\pi} \int_0^{1} N(E, p) \sqrt{1 - p^2} dp.
\]

For the vortex, in Eqs. (17) and (18), \( E \rightarrow E + E_0 \) sign \( E \), \( E_0 = \hbar |p_i| v/2\Delta_0 r_c^2 \), Eq. (15), with \( p_i \) from Eq. (16), and \( \sum_{p_i} \) instead of integration in Eq. (19). Here, signs of \( p \) and \( E \) are connected, because the magnetic field causes a difference in propagation of particles and holes.
For small radius vortices in HTS, $\xi_0 mv/\hbar \sim 1$ ($\sim 10$ in classical superconductors), only one trajectory through the vortex core is allowed, with $p = p_0$, Eq. (16). Taking for YBCO $r_c = \xi_0$, $p_0 = 1/3$, $\Delta_0/E_F = 0.424$, only one peak in DOS in the vortex core around $E/\Delta_0 \approx 0.3$ is obtained (Fig. 2). For comparison, DOS of normal metal cylinder embedded in the same superconductor and with the same radius $r_c = \xi_0$, but in the zero magnetic field, is shown. In this case, a large energy gap is found in DOS, due to formation of lowest bound state at high energy, of the order of $\Delta_0$. This is not the case in classical superconductors, where $\Delta_0/E_F \ll 1$.

In conclusion, cylindrical Fermi surface, $d$-wave pairing and small $\xi_0$, large $\Delta_0/E_F \sim 0.1$, make DOS of a normal metal cylinder embedded in HTS and a pancake vortex different from DOS of a normal cylinder and a vortex in classical superconductors. Since the Andreev bound states can transport charge currents, unlike the bound states in a potential well, supercurrents can flow through the vortex without losses, strongly influencing its dynamics. This could be very important for transport properties of HTS, especially for understanding the unusual magnetic-field dependence of the electrothermal conductivity, which was observed experimentally and awaits explanation.

**References**
